Optimum Take-Off Techniques for High and Long Jumps

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Optimum take-off techniques for high and long jumps

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SUMMARY

High jumpers run at moderate speeds and set down the foot, from which they take off, well in front of the body. Long jumpers run up much faster and place the foot less far forward, with the leg at a steeper angle. A simple model, which takes into account the mechanical properties of muscle, predicts optimum take-off techniques that agree well with those used by athletes. These predictions are remarkably insensitive to the numerical values assigned to the physiological parameters.

1. INTRODUCTION

Athletes taking off for a high or long jump set a foot down on the ground well in front of the body with the leg almost straight. The knee flexes and then extends, before the foot leaves the ground for the jump. These movements have been discussed in many papers, for example, Hay (1986) and Dapena & Chung (1988), but there seems to be no theory of optimum take-off technique. High jumpers enter the final step of their run up with their centre of mass lower than long jumpers so that the leg makes a smaller angle with the horizontal when the foot is first set down. It seems obvious that they should do this, to throw the body more steeply into the air, but there is no theory to predict the optimum angle at which the leg should be set down. High jumpers enter the final step at much lower speeds than long jumpers: for world-class male athletes the speeds are about 7 m s⁻¹ for the high jump and over 10 m s⁻¹ (close to peak sprinting speeds) for the long jump. It is not obvious whether high jumpers have to run more slowly to maintain adequate control of take-off, or whether the lower speed gives some other advantage. My aim in this paper is to identify the principles that govern optimum speed and leg angle, for the take-off both of high jumping and of long jumping. I use a simple model that considers the properties of the leg muscles.

2. THE MODEL

The model (figure 1 *a*) resembles an earlier model of standing jumps (Alexander 1989). It is very simple (much simpler, for example than the model proposed by Hatze (1983)), but will be shown in later sections to predict realistic force patterns and jump performance. It has a rigid trunk, and a leg formed from two rigid segments each of length *a*. The mass *m* is concentrated in the trunk and the centre of mass is at the hip joint. (The centre of mass of a man or woman, standing erect with the arms by the sides, is about 5 cm above the centres of the hip joints (Dyson 1973).)

The ground coincides with the X axis of a Cartesian

system. The foot (treated as a point at the distal end of the distal leg segment) is set down at the origin. While it is on the ground, the extensor muscles exert a torque *T* at the knee joint but there is no torque at the hip. Consequently, the ground force *F* is aligned with the hip and the centre of mass.

At time *t*, the centre of mass is at (x, y), and the line from the origin to the centre of mass makes an angle θ with the horizontal.

$$\theta = \arctan{(-y/x)}. \tag{1}$$

The angle of the knee joint is ϕ so the distance of the centre of mass from the origin is given by

$$(x^2 + y^2)^{\frac{1}{2}} = 2a \sin{(\phi/2)}. \tag{2}$$

The ground force *F* is related to the knee torque *T* by the equation

$$F = T/(a \cos{(\phi/2)}). \tag{3}$$

Movements of the knee are accompanied by changes in length both of the contractile component of the knee extensor muscles and of the series elastic component. Any change $\Delta\phi$ in knee angle is the sum of changes $\Delta\phi_c$ due to the contractile component and $C \cdot \Delta T$ due to the series elastic component, which confers angular compliance *C*

$$\Delta\phi = \Delta\phi_c + C \cdot \Delta T. \tag{4}$$

I assume that the knee extensor muscles are fully activated throughout the time that the foot is on the ground. Aura & Viitasalo (1989) found that electrical activity in the quadriceps (and gastrocnemius) muscles of high jumpers reached a high level 40 ms before the foot was set down for the take-off step and rose only a little further while the foot was on the ground. Kyröläinen *et al.* (1989) made similar observations for a long jumper, but found that high levels of activity were attained 60 ms before the foot was set down.

It seems convenient to write equations that involve torques and joint angles, rather than muscle forces and lengths (Hof & Van den Berg 1981). Assuming the foot is on the ground, the torque at the knee depends on the rate of change of length of the contractile component of

Phil. Trans. R. Soc. Lond. B (1990) **329**, 3–10   *Printed in Great Britain*

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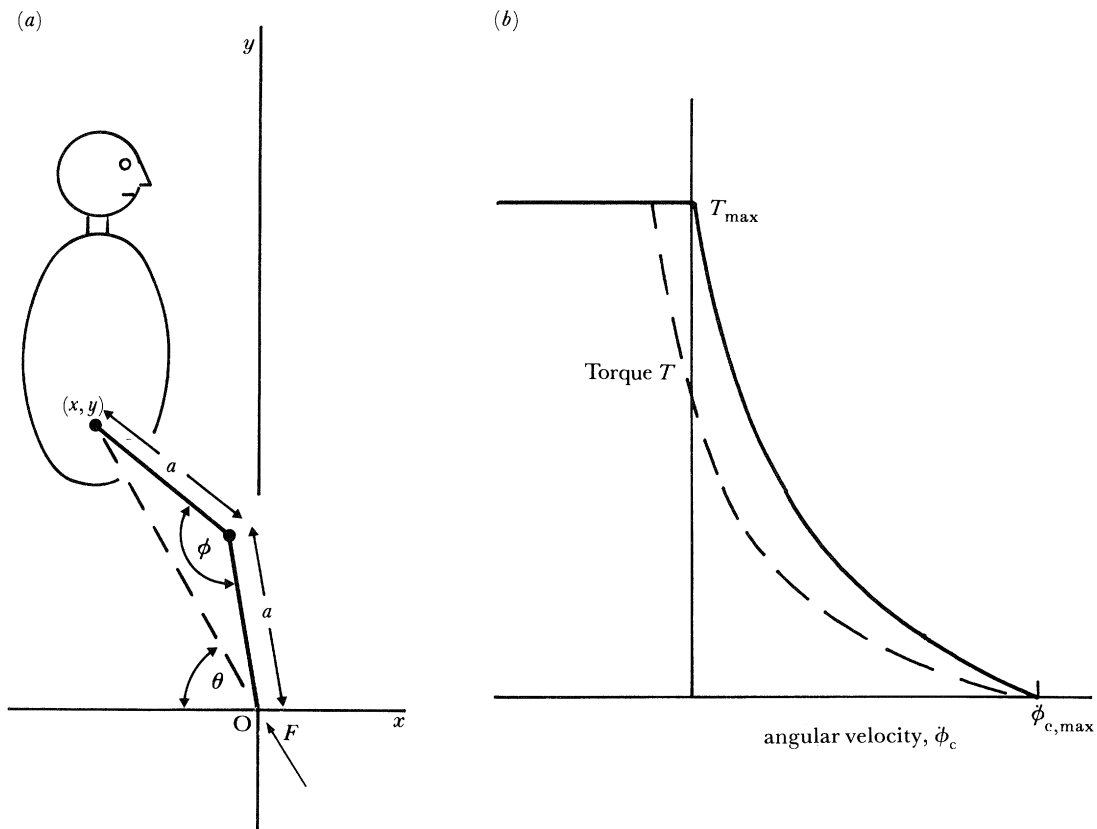(a)   (b)

Figure 1. (a) A diagram of the model. (b) A graph showing the assumed relation between the rate of shortening of the contractile component of the knee muscles and the torque T that they exert. The rate of shortening is represented by the corresponding rate of change of knee angle, in the absence of series compliance. The continuous line represents equations (5a) and (5b). The broken line is explained in §4.

the knee muscles according to the equations (figure 1b):

$$\text{if} \quad \dot{\phi}_{c} \leqslant 0, \quad T = T_{max}, \tag{5a}$$

$$\text{if} \quad \dot{\phi}_{c} > 0, \quad T = T_{max}[(\dot{\phi}_{c,max} - \dot{\phi}_{c})$$

$$/(\dot{\phi}_{c,max} + G\dot{\phi}_{c})]. \tag{5b}$$

Here T_{max} is the maximum torque exerted by the muscles in eccentric activity (i.e. when they are being stretched forcibly), $\dot{\phi}_{c,max}$ is the angular velocity of the knee corresponding to the muscle's unloaded rate of shortening, and G is a constant. Equation (5a) seems justified by experiments on isolated frog leg muscles that exerted 1.4 to 1.8 times their isometric forces while being stretched at all but the lowest rates (Woledge *et al.* 1985). Equation (5b) is a version of Hill's muscle equation (see, for example, Woledge *et al.* (1985)), with the maximum torque attained in eccentric activity (T_{max}) used instead of the isometric torque because muscles exert enhanced forces when shortening after a rapid stretch (Cavagna *et al.* 1968).

While the foot is on the ground, the equations of motion of the centre of mass are:

$$\ddot{x} = -(F/m) \cos \theta, \tag{6}$$

$$\ddot{y} = (F/m) \sin \theta - g, \tag{7}$$

where g is the gravitational acceleration. The foot leaves the ground when the torque (given by equation (5b)) falls to zero. At this instant (time t_{off}) the centre

of mass is at (x_{off}, y_{off}) and the components of its velocity are $\dot{x}_{off}, \dot{y}_{off}$. During the aerial phase of the jump it continues with the same horizontal component of velocity but has vertical acceleration $-g$. It rises to a maximum height h (figure 2).

$$h = y_{off} + \left(\frac{\dot{y}_{off}^{2}}{2g}\right). \tag{8}$$

Its trajectory intersects the ground at $(x_{off} + s, 0)$. To calculate the distance s (figure 2) imagine that the jump ended only when the centre of mass hit the ground. The duration of the flight through the air, Δt_{air}, would be given by

$$-y_{off} = \dot{y}_{off} \Delta t_{air} - \tfrac{1}{2}g(\Delta t_{air})^{2}.$$

The positive solution of this equation is

$$\Delta t_{air} = \frac{[\dot{y}_{off} + (\dot{y}_{off}^{2} + 2gy_{off})^{\frac{1}{2}}]}{g},$$

hence,

$$s = \dot{x}_{off} \Delta t_{air}$$
$$= (\dot{x}_{off}/g) \ [\dot{y}_{off} + (\dot{y}_{off}^{2} + 2gy_{off})^{\frac{1}{2}}]. \tag{9}$$

Aerodynamic drag has been ignored in the above equations because a rough calculation using the data of Ward-Smith (1986) shows that its effect would be small.

I have used equations (1–9) in numerical simulations of jumps, run on a microcomputer, to obtain the results
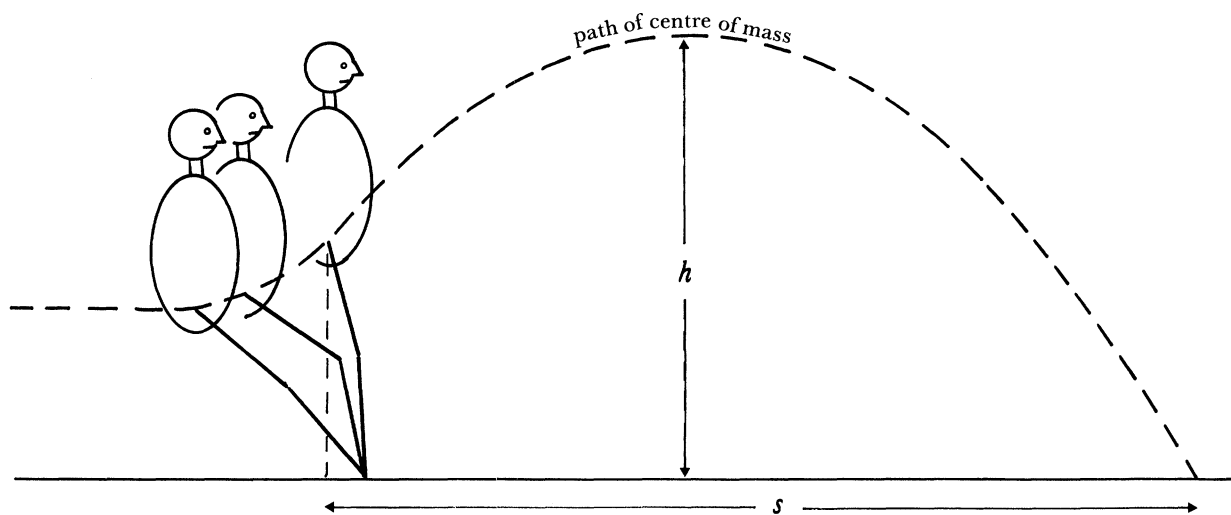
Figure 2. Successive positions of the model during a jump. The position marked 'on' occurs at the instant when the foot is set down, and 'off' at the instant when it leaves the ground.

presented in this paper. The next section explains my choice of values for the quantities used in the equations.

3. ASSIGNMENT OF VALUES TO PARAMETERS

Because athletes differ from each other in mass and stature, to make the calculations as generally applicable as possible, dimensionless quantities have been used, expressing forces as multiples of the weight mg of the body, distances as multiples of the leg segment length a and times as multiples of $(a/g)^{\frac{1}{2}}$. When the model stands with its legs straight and vertical, its centre of mass is at a height $2a$ above the ground. When people stand similarly, the height of the centre of mass is 55 % of their stature (Dyson 1973). The leg segment length a can therefore be estimated as 28 % of stature, for example 0.50 m for a man 1.80 m tall. In calculating values for a typical male athlete I will assume $a = 0.50$ m and body mass $m = 70$ kg.

For the maximum torque T_{max}, I have chosen a value of 2.5 mga (860 Nm^{-1} for a typical male athlete). This is probably larger than the torques that act at the knees of athletes because the model locates the ground force at the distal end of its tibia whereas the ground forces on real athletes presumably act further forward, on the ball of the foot: consequently, the model needs an unrealistically large knee moment to give a realistic ground force. It is shown in §4 that the chosen maximum torque gives realistic ground forces F. I have given G (equation (5b)) a value of 3, a typical value for the corresponding Hill's equation parameter for fast mammalian muscle (Woledge *et al.* 1985). (G is the reciprocal of the parameter for which they use the symbol a/P_0). It seems more difficult to identify an appropriate value for the quantity $\dot{\phi}_{\mathrm{c,max}}$ that describes the intrinsic speed of the muscles. The peak angular velocity attained by the knee during take off for standing jumps is about 17 rad s^{-1} (Bobbert & van Ingen Schenau 1988). For a man of leg segment length 0.5 m, this is 3.8$(g/a)^{\frac{1}{2}}$. It seems likely that the unloaded rate of shortening of the quadriceps muscles is higher

than this, and I have used 8$(g/a)^{\frac{1}{2}}$ for most of my calculations. I also present results for $\dot{\phi}_{\mathrm{c,max}} = 20(g/a)^{\frac{1}{2}}$ to test the sensitivity of the model to this doubtful parameter. It also seems difficult to select a realistic value for the series elastic compliance C, which probably reflects the properties of the tendons more than those of the muscles (Alexander & Bennet-Clark 1977). I present results for $C = 0$ and $C = 0.1 \ (mga)^{-1}$. With the latter value, the maximum torque 2.5 mga would stretch the series elastic component by an amount corresponding to 0.25 rad (14°) movement of the knee, storing strain energy ($\frac{1}{2}T_{\mathrm{max}}^2 C$) amounting to 0.31 mga, or 106 J for a typical male athlete. Ker *et al.* (1987) estimated that about 52 J strain energy is stored during a running step, in the Achilles tendon and the ligaments of the arch of the foot, and additional energy must be stored in the patellar tendon. Larger forces act in long jumping than in running, and more strain energy must be stored.

The initial values of variables (their values at the instant when the foot is set down on the ground) are distinguished by the subscript 'on'. I assume $\dot{y}_{\mathrm{on}} = 0$ because the vertical component of the initial velocity of the centre of mass is much smaller than the horizontal component, both in high jumping and in long jumping (Dapena & Chung 1988; Hay 1986). I will investigate the effects on jumping performance of different values of the initial horizontal velocity $\dot{x}_{\mathrm{on}}$ and the initial leg angle θ_{on}. The peak speeds attained in springing races by world-class male athletes are about 11 m s^{-1} (Baumann *et al.* 1986), or about 5 $(ag)^{\frac{1}{2}}$ for $a = 0.5$ m, so speeds only up to this limit will be investigated.

The muscles are assumed to be maximally active from the instant when the foot hits the ground, but the initial torque T_{on} depends on the initial state of the series elastic component. If the muscles had been inactive until that instant, the series elastic component would be unstrained and T_{on} would be zero. It seems more likely that the muscles develop tension before the foot is set down, but this is likely to be their isometric tension rather than the higher tension that they can exert only in eccentric activity. I have therefore
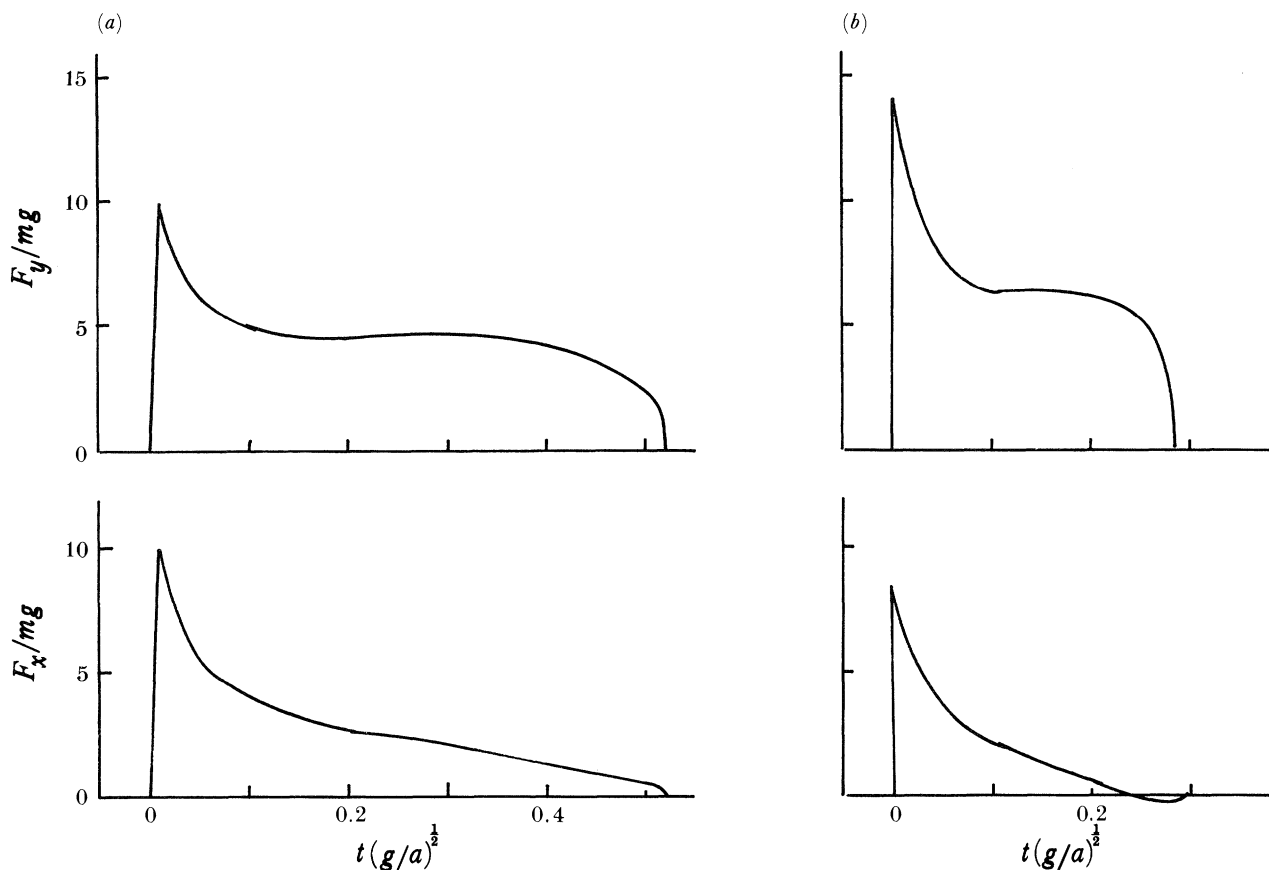
Figure 3. The forces exerted by the model on the ground in simulations of (a) a high jump and (b) a long jump. The vertical and horizontal components of force are shown as multiples of body weight $\left(\dfrac{F_y}{mg} \text{ and } \dfrac{F_x}{mg}, \text{ respectively}\right)$.

They are plotted against time, which is represented by the dimensionless group $t(g/a)^{\frac{1}{2}}$. In both cases $T_{\max} = 2.5 \, mga$, $\dot{\phi}_{c,\max} = 8(g/a)^{\frac{1}{2}}$, $C = 0.1/mga$, $G = 3$, $\phi_{on} = 170°$ and $T_{on} = 0.6 \, T_{\max}$. In (a), $\dot{x}_{on} = 3(ag)^{\frac{1}{2}}$, $\theta_{on} = 45°$. In (b), $\dot{x}_{on} = 4.5(ag)^{\frac{1}{2}}$, $\theta_{on} = 60°$.

assumed $T_{on} = 0.6 \, T_{\max}$ in most of the calculations involving a series elastic compliance. When there is no series compliance ($C = 0$), $T_{on} = T_{\max}$. If the leg was initially straight, equation (3) would give an infinite force F for any non-zero value of T_{on}. I have avoided this by assuming an initial knee angle (ϕ_{on}) of 170°.

4. RESULTS

Figure 3 shows examples of patterns of force exerted by the model on the ground. Figure 3a simulates a high jump. The initial speed of the model was $3(ag)^{\frac{1}{2}}$ (6.6 m s^{-1} for a typical male athlete) and it set its leg down at an angle (θ) of 45°. Similarly, the high jumpers studied by Dapena & Chung (1988) had a mean speed of 6.7 m s^{-1} at the start of the take-off phase and set down their legs at a mean of 47°. When the foot left the ground the centre of mass of the model was travelling at 49° to the horizontal, and those of the athletes at a mean of 44°. The centre of mass of the model rose to a height of 3.75a above the ground (1.88 m for a typical athlete, enough to clear about that height in a Fosbury flop in which the centre of mass *may* pass slightly under the bar (see Müller 1986). The athletes cleared a mean height of 2.04 m.

The vertical force calculated for the model (figure 3a) shows an initial peak of ten times body weight

(7000 N for a 70 kg athlete) followed by a plateau at about 4.5 times body weight (3100 N). Similarly, a force record of take-off by a high jumper (Aura & Viitasalo (1989), subject A) shows an initial peak of 8.4 times body weight followed by a plateau at about 4.9 times body weight.

In all these respects the simulated jump is reasonably realistic, but in one important respect it is not. The foot remained in contact with the ground for a time of only $0.52(a/g)^{\frac{1}{2}}$ (about 120 ms for our typical male athlete), but the real athletes maintained contact with the ground for 155 ms (Aura & Viitasalo 1989) or 185 ms (Dapena & Chung 1988). The discrepancy must be due at least in part to the model's lack of a foot. The toes of high jumpers remain on the ground for some time after the heel has left it. Also, the force exerted by the model rises unrealistically rapidly to its initial peak because no account is taken of the elastic properties of the foot and shoe.

Figure 3b simulates a long jump. The initial speed of the model was $4.5(ag)^{\frac{1}{2}}$ (10.0 m s^{-1} for a typical male athlete) and the leg was set down at 60° to the horizontal. The 'national level' long jumpers studied by Luhtanen & Komi (1979) ran up at a mean speed of 9.6 m s^{-1}. Long jumpers set down the foot at the beginning of the take-off phase with the line from the centre of mass of the body to the heel inclined at 64–69°
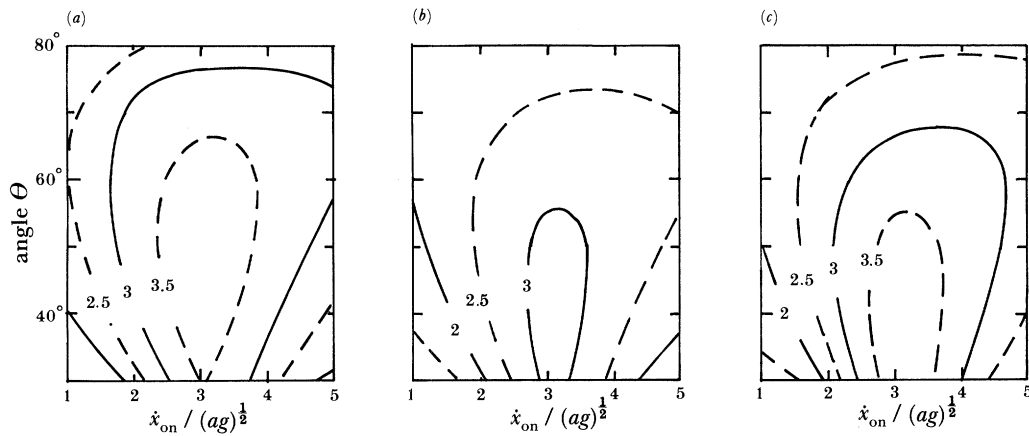
Figure 4. Graphs of initial knee angle (θ_{on}) against initial speed $\left(\text{represented by the dimensionless group } \dfrac{\dot{x}_{on}}{(ag)^{\frac{1}{2}}}\right)$, with

contours showing the relative height (h/a) of the jump. In every case $T_{max} = 2.5\ mga$, $G = 3$, $\phi_{on} = 170°$

In (a), $\dot{\phi}_{c,max} = 8(g/a)^{\frac{1}{2}}$, $C = \dfrac{0.1}{mga}$,

$\qquad T_{on} = 0.6\,T_{max}$

In (b), $\dot{\phi}_{c,max} = 8(g/a)^{\frac{1}{2}}$, $C = 0$,

$\qquad T_{on} = T_{max}$

In (c), $\dot{\phi}_{c,max} = 20(g/a)^{\frac{1}{2}}$, $C = 0$,

$\qquad T_{on} = T_{max}$

to the horizontal (Hay 1986): a line through centre of mass and ankle, which is more closely comparable to the line of the model's leg, would be inclined at about 60–65°. When the foot left the ground the centre of mass of the model was travelling at 22° to the horizontal, a slightly larger angle than the mean of 20° observed by Luhtanen & Komi (1979), but well within the normal range (Hay 1986). The centre of mass of the model travelled a horizontal distance of 16.5 a (8.3 m for our typical athlete) between its position at the instant when the foot left the ground and the intersection of its trajectory with the ground. This probably corresponds to an official jump distance of at least 7.5 m, rather more than the mean of 7.0 m recorded by Luhtanen & Komi's (1979) athletes, but well within the range expected in competition. (The position of the leg at take-off would probably have put the centre of mass almost directly over the edge of the board, but recorded landing points are a little behind the theoretical point of impact of the centre of mass (Hay 1986)).

The calculated force patterns are reasonably realistic in shape (compare with those in Kyröläinen *et al.* (1989)), except that the force rises too fast to its initial peak. However, the forces are rather larger than for real athletes and the duration of the foot contact shorter. The initial peak of 14 times body weight and the plateau at about 6.3 times body weight, in the vertical component of force, are larger than the corresponding values of 11 and 4.7 times body weight recorded by Luhtanen & Komi (1979). The duration of the foot contact is $0.28(a/g)^{\frac{1}{2}}$ for the model (corresponding to 64 ms for our typical athlete), but Luhtanen & Komi (1979) recorded a mean ground contact time of 110 ms for their national level athletes.

The fast initial rise in force and the short duration of ground contact are presumably largely because of the model's lack of a foot, as already explained for the high jump. The forces are larger in the simulated long jump than in the high jump, for the same torque, because the knee bends less, to a minimum of 135° instead of 123°.

These examples show that the model, though simple, is capable of simulating high and long jumping reasonably well. I now investigate its behaviour more systematically.

Figure 4 shows, by means of contours, the heights to which the model's centre of mass would rise in jumps that use different initial speeds and leg angles. In figure 4a the other parameters of the model are the same as those in figure 3. The highest jumps are obtained with an initial speed of about $3(ag)^{\frac{1}{2}}$ and an initial leg angle of 45–50°, that is, at about the speed and leg angle used for the example of a high jump shown in figure 3a. Notice that a faster run-up does not necessarily give a higher jump: the highest jumps are obtained at intermediate speeds.

It seems pertinent to ask whether this conclusion depends critically on the values assigned to the parameters that were kept constant in figure 4a. In b, the maximum torque and the intrinsic speed of the muscles are the same as before, but the series compliance has been reduced to zero. The highest jumps are still obtained at a speed of about $3(ag)^{\frac{1}{2}}$, but the optimum leg angle is slightly lower than before (about 40°) and the greatest obtainable height is only 3.2 a (about 1.6 m for our typical athlete) instead of the more realistic 3.8 a. In figure 4c the compliance is again zero, but the height of the best jump has been restored by increasing the intrinsic speed of the muscles to a value that is probably unrealistically high. The

optimum leg angle is even lower than in (*b*), but the highest jumps are still obtained with an initial speed of $3(ag)^{\frac{1}{2}}$.

The calculation of figure 4*a* was repeated with different initial knee angles. The highest jump obtainable was about 3.6 *a* for an initial knee angle of 160° and 3.9 *a* for 178°, but the changes had little effect on optimum speed and leg angle. An initial knee angle of 178° gave unrealistically high impact forces. Increasing the maximum torque from 2.5 to 3.8 *mga*, with other parameters shown in figure 4*b*, shifted the optimum to a higher speed $(4(ag)^{\frac{1}{2}})$ and a larger initial leg angle (55°) and gave jump heights up to 3.9 *a*. However, with zero compliance even this very high torque (which gave unrealistically high ground forces) gave a poor jump of only 3.5 *a* when the initial speed and leg angle were adjusted to more realistic values of $3(ag)^{\frac{1}{2}}$ and 45°. Changing the Hill's equation parameter *G* from 3 to 5 (making the force–velocity curve more concave) reduced the height of jumps a little, but left the maximum at the same speed and angle as in figure 4*b*. The results for the model with series compliance (figure 4*a*) assume an initial torque of 0.6 T_{max}. Reducing this initial torque to zero makes the force patterns unrealistic, eliminating the initial peak.

The initial vertical component of velocity, $\dot{y}_{\mathrm{on}}$, was assumed to be zero in most of the simulations, but might more realistically have been given a small negative value (Dapena & Chung 1988). I therefore repeated the calculations of figure 4*a* with $\dot{y}_{\mathrm{on}} = -0.2(ag)^{\frac{1}{2}}$ (-0.44 m s^{-1} for our typical athlete). This reduced the highest obtainable jump by 0.2 *a* but had no appreciable effect on the optimum run-up speed and initial leg angle.

In another simulation, I modified the properties of the muscle as indicated by a broken line in figure 1*b*, thus eliminating the effect described by Cavagna, Dusman & Margaria (1968). (The broken line has been obtained by multiplying the right hand side of equation 5*b* by 0.6.) With the same parameter values as in figure 4*a*, the highest obtainable jump was reduced

to 3.4 *a* and the optimum speed and angle became $3(ag)^{\frac{1}{2}}$ and 55°.

Figure 5 shows the lengths of jumps, for different initial speeds and leg angles. In 5*a* the other parameters are the same as for the jumps of figure 3. The longest jump is obtained at the highest speed, with an initial leg angle of 70°. The highest speed shown in the figure would correspond to an exceptional sprinting speed of 11 m s^{-1} (for our typical athlete): the graph suggests that long jumpers, unlike high jumpers, should make their run-up as fast as possible consistent with planting the leg at the appropriate angle. It also shows that the optimum initial leg angle is larger than for high jumping.

Figure 5*b* shows that, without series compliance, the model cannot jump so far, but the longest jumps are still obtained at the highest speed and with a steep leg angle. Figure 5*c* shows that muscles of higher intrinsic speed (still with no series compliance) make longer jumps possible, but the optimum is still at high speed and steep leg angle.

5. DISCUSSION

Much of the skill of high and long jumping consists in controlling the orientation of body segments in the air, so as to clear the highest possible bar or make the rearmost mark on landing as far forward as possible for a given trajectory of the centre of mass (Müller 1986; Hay 1986). These subtleties are ignored in this paper, which is concerned only to find the take-off technique that optimizes the trajectory.

The model is grossly simplified. It has no segment corresponding to the foot, and no foot compliance: consequently, the force at impact rises unrealistically rapidly and the duration of ground contact is unrealistically short, as already noted. Its leg segments have no mass: if they had, the force required to decelerate them would contribute to the ground force following impact (see Ker *et al.* (1989), on impact forces in running). The centre of mass is assumed to
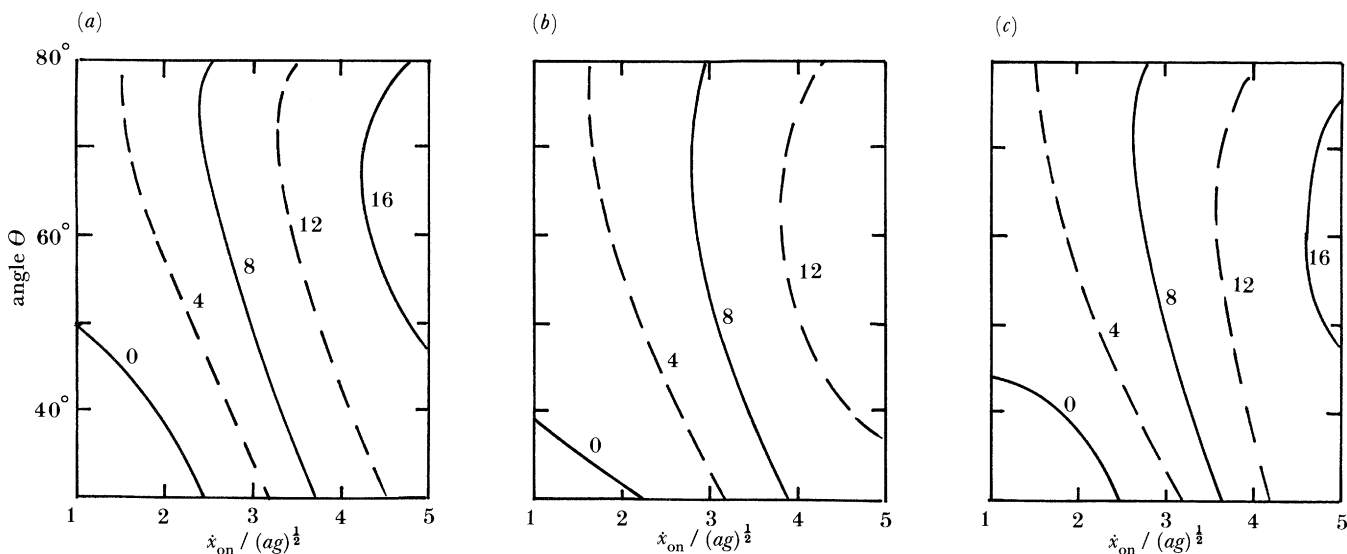

Figure 5. As for figure 4, but with contours showing the relative length $\dfrac{s}{a}$ of the jump. The parameters were assigned the same values, for (*a*), (*b*) and (*c*), as for the corresponding graphs in figure 4.

coincide with the hip joint. Only one muscle (regarded as a knee extensor) is included, and it is assumed to be fully active throughout the period of ground contact. The torque that this muscle can exert (for a given rate of change of knee angle) is assumed to be independent of knee angle, although the effective moment arm of the human quadriceps group decreases as the knee flexes (Lindahl & Movin 1967) and the forces that muscles can exert depend on their current sarcomere length (see Woledge *et al.* 1985). It seemed justifiable to ignore the dependence of torque on knee angle because the model uses a restricted range of knee angles, for example, 123–170° in the high jump shown in figure 2*a* and 135–171° in the long jump shown in figure 2*b*. The advantage of using such a simple model is that complexity tends to obscure basic principles.

Many of the values needed for parameters in the model are doubtful. Those used for the intrinsic speed of the muscles and for the series compliance are little better than guesses. The maximum torque was chosen so as to obtain reasonably realistic ground forces, and is not intended to match the torques that act at the knees of athletes, as explained in §3. With the parameters used in figure 3, the model gives fairly realistic patterns of ground force and realistic heights and lengths of jump, for appropriate initial speeds and leg angles. This suggests that the chosen values are not far wrong.

6. CONCLUSION

The main conclusions from the model are affected only a little by large changes in the doubtful parameters. A high jumper should run up at a moderate speed (about $7~\mathrm{m~s^{-1}}$ for a typical male athlete) and set down the foot from which he takes off with the leg at an angle of about 45°: this shallow angle is achieved by running up with the centre of mass low, and setting down the foot well in front of the body (Dapena & Chung 1988). A long jumper should run up as fast as possible and set down the leg at a steeper angle. High and long jumpers have, of course, learned by experience that these are the best ways to jump. The main interest of this paper is that a simple model based on our knowledge of muscle physiology has sufficed to explain why these techniques are best. It was not obvious (at least to me) that the relatively low run-up speeds used by high jumpers gave any advantage other than making it easier to control take-off and to obtain the complicated movements required in the aerial phase of the jump.

The conclusions come most clearly from the mathematics, but it is possible to explain the underlying principles in words. To obtain the upward momentum required for a jump, an athlete must exert a downward impulse on the ground. The forces that can be exerted are limited by the properties of the leg muscles so the impulse (force integrated over time) depends critically on the duration of foot contact. A fast run-up makes for a large horizontal component of velocity at take-off, but shortens the duration of ground contact and hence restricts the vertical impulse. The horizontal component of velocity at take-off is more important in long

jumping than in high jumping, so a faster run-up is desirable in long jumping. Similarly, a shallow initial leg angle makes it possible for the foot to be kept on the ground while the body travels a longer distance, and so to increase the duration of foot contact and the vertical impulse. However, when the angle is shallow, the (desirable) vertical component of ground force is accompanied by an (undesirable) horizontal component, which reduces the horizontal component of the take off velocity. This component of velocity is more important in long jumping than in high jumping, so a long jumper should set down his leg at a steeper angle than a high jumper. It might be thought that a horizontal decelerating impulse delivered early in the take-off phase could be counteracted by an accelerating impulse after the leg has passed the vertical: this happens in each step when a person runs at a steady speed (see Alexander 1984). However, if it is necessary to activate all the muscle early in the period of ground contact, to obtain the required impulse, it will exert much larger forces (while being stretched) than it can while shortening later in the step. For this reason, the muscles do net negative work during take-off, reducing the total (kinetic plus potential) energy of the body. This is a major difference between high jumping and pole vaulting, in which the potential energy gained is approximately equal to the kinetic energy lost.

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SYMBOLS USED IN THE TEXT AND FIGURES

a the length of each leg segment (figure 1 a)

C the angular compliance due to the series elastic compliance of the knee extensor muscles

F the force exerted by the foot on the ground

g the acceleration of free fall

h the height to which the centre of mass rises in a jump (figure 2)

m the mass of the body

s the horizontal distance travelled by the centre of mass, from the instant when the foot leaves the ground to the intersection of its trajectory with the ground (figure 2)

T the torque exerted by the knee extensor muscles

t time

x, y the coordinates of the centre of mass (figure 1 a)

θ the angle between the line from hip to foot, and the ground (figure 1 a)

ϕ the angle of the knee joint (figure 1 a)

subscript 'air' refers to the aerial phase of the jump

subscript 'c' refers to the contractile component of the muscle

subscript 'max' indicates the maximum possible value

subscript 'off' refers to the instant when the foot leaves the ground

subscript 'on' refers to the instant when the foot is placed on the ground

subscripts 'x', 'y' refer to the x and y directions (figure 1 a)